Lexicase Selection

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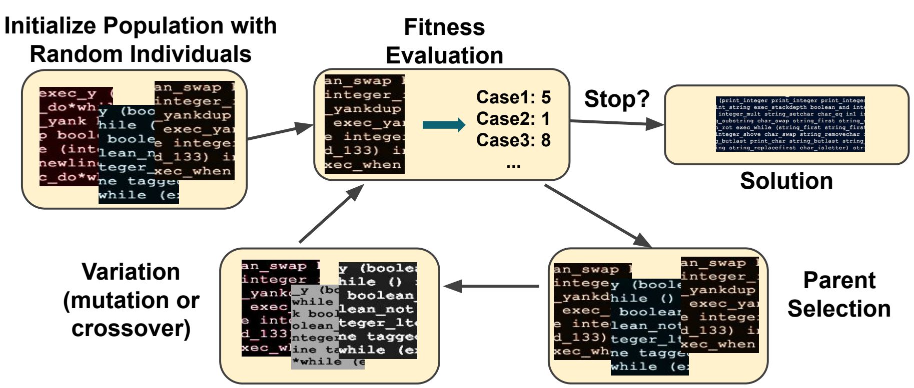
http://gecco-2021.sigevo.org/

GECCO '21 Companion, Lille, France

ACM ISBN 978-1-4503-8351-6/21/07 $15.00
https://doi.org/10.1145/3449726.3461408

Background and Motivation

Parent Selection in Evolutionary Computation



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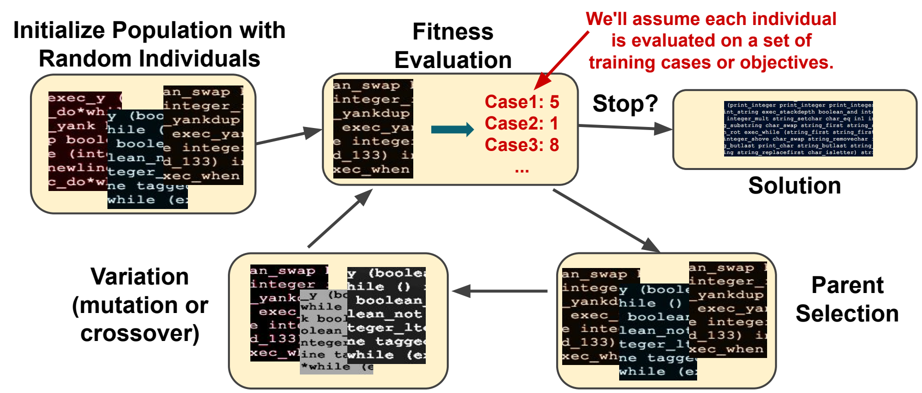

Nomenclature

- (training) **Cases**:
  - Samples of training data
  - Sometimes referred to as "test cases"
- Semantics:
  - The behavior of a GP program on the training cases
  - The genome of a GA
- Errors:
  - The (absolute, squared etc.) difference between an individual's semantics and the desired semantics on the training cases

	Training Data				
Cases	x1	x2	x3	x4	Target
1	1	0	86	7.5	6
2	0	1	3	6.9	3
3	1	3	45	12.3	8
4	1	6	-6	0.78	9
5	0	5	29	1.2	2

Individual A	
Case	Semantics
1	16
2	8
3	13
4	-6
5	12

	Individual Errors				
Case	A	B	C	D	E
1	10	8	73	15	15
2	5	7	60	12	12
3	5	8	0	14	0
4	15	8	0	15	106
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Total Error:	45	38	134	57	134

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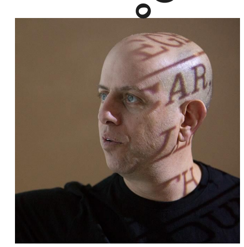
Origin Story

- Late one night...
- How can Calculator Behavior be evolved?
  - Multiple unrelated functions
    - Different test cases
  - How to maintain behaviors that are good on problem subsets?

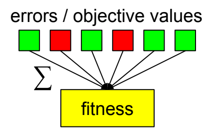


Lee Spector

Spector, Lee. (2012). Assessment of problem modality by differential performance of lexicase selection in genetic programming: a preliminary report. *GECCO.*

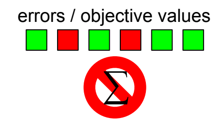
Motivations

- Most parent selection methods use a single *aggregated* fitness value
  - Ex: total error across set of training cases
  - Even multi-objective methods (e.g. NSGA-II) and quality diversity methods aggregate errors
- Obscures useful info
  - Ex: Individual Q performs well on some cases and poorly on others
    - perhaps Q has genetic material worth propagating!
    - Q has poor total error
    - Q not likely selected by tournament selection
      - The skill Q is good at may be lost in the population
- Generalists vs. Specialists



Motivation: Semantic-Aware Selection

- De-aggregating fitness
- Aggregating creates an "Information Bottleneck" - taking a rich amount of information in errors and reducing it to a single value
  - Krawiec
- Semantic-aware selection methods make use of all semantics/errors



- Krawiec, K., et al. (2015). Behavioral Program Synthesis: Insights and Prospects. *GPTP*
- Krawiec, K., & O'Reilly, U.-M. (2014). Behavioral Programming: A Broader and More Detailed Take on Semantic GP. *GECCO*.

When it's applicable

- When *fitness* can be decomposed into component parts.
  - Summations / averages over cases (mean squared error, etc)
- Places it doesn't apply:
  - Single output, black-box function optimization
- There are *enough* component parts of fitness
  - There are factorial(n_cases) ways to be selected with lexicase selection

Who is rewarded

- Individuals that are good at cases that others aren't good at
  - More cases ✅
  - More difficult cases ✅

Example Uses of Lexicase Selection

GP Program Synthesis

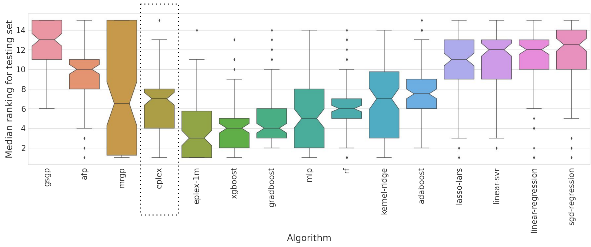
- Program synthesis: generating programs with multiple data types and control flow structures
- Lexicase selection has outperformed tournament selection and other selection methods across many benchmark problems

- Thomas Helmuth and Lee Spector. (2015) General program synthesis benchmark suite. *GECCO*
- Forstenlechner, S. et al. (2017). A Grammar Design Pattern for Arbitrary Program Synthesis Problems in Genetic Programming. *EuroGP*.

Problem	Tourn	IFS	Lex
Number IO	68	72	98
Small Or Large	3	3	5
For Loop Index	0	0	1
Compare String Lengths	3	6	7
Double Letters	0	0	6
Collatz Numbers	0	0	0
Replace Space with Newline	8	16	51
String Differences	0	0	0
Even Squares	0	0	2
Wallis Pi	0	0	0
String Lengths Backwards	7	10	66
Last Index of Zero	8	4	21
Vector Average	14	13	16
Count Odds	0	0	8
Mirror Image	46	64	78
Super Anagrams	0	0	0
Sum of Squares	2	0	6
Vectors Summed	0	0	1
X-Word Lines	0	0	8
Pig Latin	0	0	0
Negative To Zero	10	8	45
Scrabble Score	0	0	2
Word Stats	0	0	0
Checksum	0	0	0
Digits	0	1	7
Grade	0	0	4
Median	7	43	45
Smallest	75	98	81
Syllables	1	7	18
Problems Solved	13	13	22

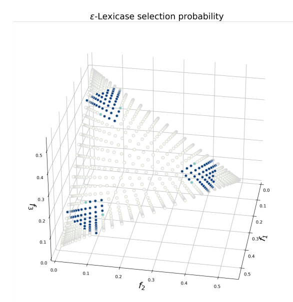
Regression

- Epsilon-lexicase selection has been shown to outperform many state-of-the-art GP and ML methods for regression

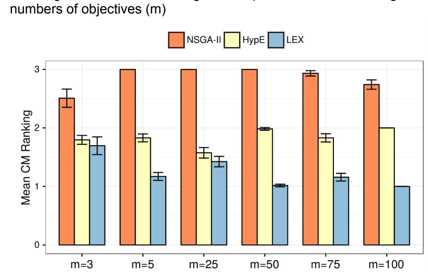


- La Cava, W. et al (2016). Epsilon-Lexicase Selection for Regression. *GECCO*
- Orzechowski, P. et al. (2018) Where Are We Now? A Large Benchmark Study of Recent Symbolic Regression Methods. *GECCO*

Many objective optimization



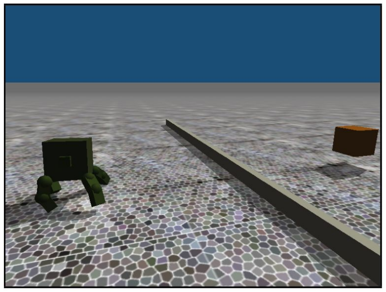
Convergence Measure Rankings, DTLZ problems, for increasing numbers of objectives (m)



La Cava, W. & Moore, J. H. (2018) An Analysis of ϵ-Lexicase Selection for Large-Scale Many-Objective Optimization. *GECCO*

Evolutionary Robotics

- In a simulated quadrupedal animat application, lexicase selection outperformed other selection methods



Moore, J. M., & Stanton, A. (2018). Tiebreaks and Diversity: Isolating Effects in Lexicase Selection. *ALIFE*.

Other Evolutionary Computation Results

- Boolean logic and finite algebras problems using GP
  - Liskowski, P. et al. (2015) Comparison of semantic-aware selection methods in genetic programming. *GECCO*.
- Learning Classifier Systems
  - Aenugu, S., & Spector, L. (2019). Lexicase Selection in Learning Classifier Systems. *GECCO*.
- Boolean constraint satisfaction using GA
  - Metevier, B. et al. (2019) Lexicase selection beyond genetic programming. *GPTP*.

The Lexicase Selection Algorithm

Lexicase Selection Algorithm:

To Pick One Parent

```
1. pool ← population
2. cases ← list of training cases, shuffled
3. while |pool| > 1 and |cases| > 0:
   a. t ← first case in cases
   b. best ← the best error value of any individual in pool on case t
   c. pool ← filter pool to include only individuals with error of best on t
   d. pop t from cases
4. if |pool| = 1:
   a. return the one individual in pool
5. else:
   a. return random individual from pool
```

Thomas Helmuth, et al. (2015) Solving uncompromising problems with lexicase selection. *IEEE Transactions on Evolutionary Computation.*

Lexicase Selection Examples

	Individual				
Case	**A**	**B**	**C**	**D**	**E**
1	10	8	73	15	15
2	5	7	60	12	12
3	5	8	0	14	0
4	15	8	0	15	106
5	10	7	1	1	1
Total Error:	**45**	**38**	**134**	**57**	**134**

Lexicase Selection: Example 1

Case order: 5, 2, 3, 1, 4

	Individual				
Case	**A**	**B**	**C**	**D**	**E**
1	10	8	73	15	15
2	5	7	60	12	12
3	5	8	0	14	0
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Total Error:	**45**	**38**	**134**	**57**	**134**

Lexicase Selection: Example 1

Case order: 5, 2, 3, 1, 4

- 5: best is 1, pool = {C, D, E}

	Individual				
Case	~~**A**~~	~~**B**~~	**C**	**D**	**E**
1	10	8	73	15	15
2	5	7	60	12	12
3	5	8	0	14	0
4	15	8	0	15	106
5	10	7	1	1	1
Total Error:	**45**	**38**	**134**	**57**	**134**

Lexicase Selection: Example 1

Case order: 5, 2, 3, 1, 4

- 5: best is 1, pool = {C, D, E}
- 2: best is 12, pool = {D, E}
  - *Note: best is always relative to pool, not full population*

	Individual				
Case	~~**A**~~	~~**B**~~	~~**C**~~	**D**	**E**
1	10	8	73	15	15
2	5	7	60	12	12
3	5	8	0	14	0
4	15	8	0	15	106
5	10	7	1	1	1
Total Error:	**45**	**38**	**134**	**57**	**134**

Lexicase Selection: Example 1

Case order: 5, 2, 3, 1, 4

- 5: best is 1,   pool = {C, D, E}
- 2: best is 12, pool = {D, E}
  - *Note: best is always relative to pool, not full population*
- 1: best is 15, pool = {D, E}

	Individual				
Case	~~A~~	~~B~~	~~C~~	**D**	**E**
1	10	8	73	15	15
2	5	7	60	12	12
3	5	8	0	14	0
4	15	8	0	15	106
5	10	7	1	1	1
Total Error:	**45**	**38**	**134**	**57**	**134**

Lexicase Selection: Example 1

Case order: 5, 2, 3, 1, 4

- 5: best is 1,   pool = {C, D, E}
- 2: best is 12, pool = {D, E}
  - *Note: best is always relative to pool, not full population*
- 1: best is 15, pool = {D, E}
- 3: best is 0,   pool = {E}
- return E

	Individual				
Case	~~A~~	~~B~~	~~C~~	~~D~~	**E**
1	10	8	73	15	15
2	5	7	60	12	12
3	5	8	0	14	0
4	15	8	0	15	106
5	10	7	1	1	1
Total Error:	**45**	**38**	**134**	**57**	**134**

Lexicase Selection: Example 2

Case order: 1, 2, 5, 4, 3

- 1: best is 8, pool = {B}
- return B

	Individual				
Case	**A**	**B**	**C**	**D**	**E**
1	10	8	73	15	15
2	5	7	60	12	12
3	5	8	0	14	0
4	15	8	0	15	106
5	10	7	1	1	1
Total Error:	**45**	**38**	**134**	**57**	**134**

Lexicase Selection: Example 3

Case order: 3, 5, 4, 1, 2

- 3: best is 0, pool = {C, E}
- 5: best is 1, pool = {C, E}
- 4: best is 0, pool = {C}
- return C

	Individual				
Case	**A**	**B**	**C**	**D**	**E**
1	10	8	73	15	15
2	5	7	60	12	12
3	5	8	0	14	0
4	15	8	0	15	106
5	10	7	1	1	1
Total Error:	**45**	**38**	**134**	**57**	**134**

Interactive Demonstration

Interactive Demonstration: Select Talk Participants

- Visit linked website to get a list of 6 random digits
  - These are your "errors" on 6 cases
- Everyone click the "raise hand" button
  - Under reactions
  - Keep your hand up as long as you're in the selection pool
- We will run through the lexicase algorithm:
  - shuffle the 6 error indices (0 through 5)
  - shrink pool, one case at a time, until one individual remains

https://www.random.org/integers/?num=6&min=0&max=9&col=100&base=10&format=html&rnd=new

Epsilon Lexicase

Working with floating point semantics

- When program semantics/errors are floating point, it is much less likely to have ties.
  - This leads to very shallow selection events using lexicase selection
- Epsilon-lexicase selection
  - Relaxes the lexicase filtering step
  - Only individuals who fall outside of some epsilon of best are filtered each step

- La Cava, W. et al (2016). Epsilon-Lexicase Selection for Regression. *GECCO*
- La Cava, W. et al. (2019). A Probabilistic and Multi-Objective Analysis of Lexicase Selection and Epsilon-Lexicase Selection. *Evolutionary Computation.*

epsilon-Lexicase Selection Algorithm:

To Pick One Parent

```
1. pool ← population
2. cases ← list of training cases, shuffled
3. while |pool| > 1 and |cases| > 0:
   a. t ← first case in cases
   b. best ← the best error value of any individual in pool on case t
   c. epsilon ← median absolute deviation of population on case t
   d. pool ← filter pool to include only individuals within epsilon of best
   e. pop t from cases
4. if |pool| = 1:
   a. return the one individual in pool
5. else:
   a. return random individual from pool
```

(static) epsilon-Lexicase Selection Algorithm:

To Pick One Parent

Calculated once per generation

```
1. pool ← population
2. cases ← list of training cases, shuffled
3. while |pool| > 1 and |cases| > 0:
   a. t ← first case in cases
   b. best ← the best error value of any individual in pool on case t
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   e. pop t from cases
4. if |pool| = 1:
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5. else:
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```

(dynamic) epsilon-Lexicase Selection Algorithm:

To Pick One Parent

Calculated dynamically

```
1. pool ← population
2. cases ← list of training cases, shuffled
3. while |pool| > 1 and |cases| > 0:
   a. t ← first case in cases
   b. best ← the best error value of any individual in pool on case t
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Optimizations and Tricks

Pre-Selection Filtering: Motivation

- In GP, programs often produce the same error vector as other programs
  - Call these *equivalent*
- If 2 or more equivalent programs would make it to the end of lexicase, we would need to look at every case to find this out
  - This is inefficient
  - If only one such individual existed, we could stop lexicase earlier

	Individual	
Case	A	B
1	17	17
2	0	0
3	4	4
4	12	12
5	1	1

Pre-Selection Filtering: Algorithm

- Group individuals into *equivalence classes* based on their error vectors
  - once per generation
- Run lexicase selection on *error vectors*, *one from each equivalence class*
  - instead of individuals
- After picking an error vector, select a random individual from its equivalence class as a parent

- This has no *functional* effect on the results of lexicase - same probability of selection for every individual
- Can provide substantial speedup of running times

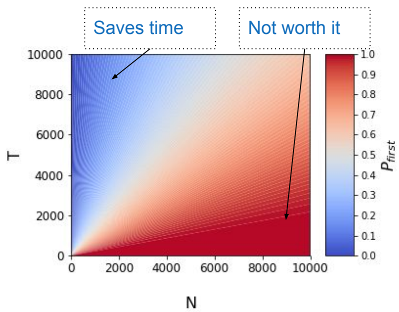
Thomas Helmuth, et al. (2020) On the importance of specialists for lexicase selection. *GPEM*

Lazy Evaluation

- Some training cases may not get used for selection
- Computational savings depend on the ratio of training cases (T) to population size (N).
- Every case probably comes first in selection when

$$T \leq \frac{1}{1-(0.5)^{1/N}}$$

- Otherwise, lazy evaluation may see significant gains in performance.



The probability of a case appearing first.

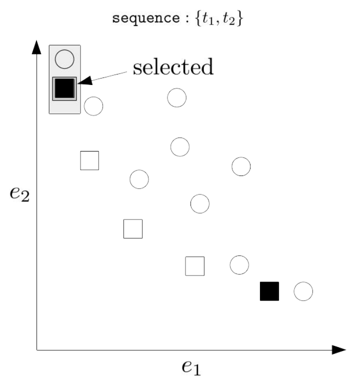
La Cava, W. et al. (2019). A Probabilistic and Multi-Objective Analysis of Lexicase Selection and Epsilon-Lexicase Selection. *Evolutionary Computation*

Multi-objective interpretations

Lexicase Selections are Pareto Optimal

- Individuals who are selected are on the Pareto front defined by the cases

Lexicase Selection



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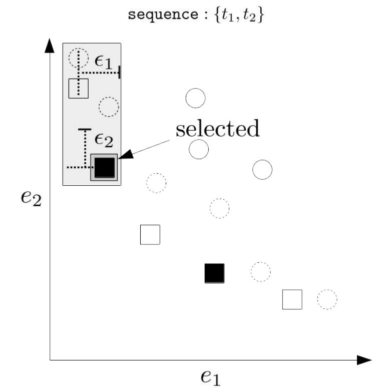
epsilon-Lexicase Selections are epsilon-Pareto Optimal

- Epsilon Lexicase selects individuals that are *epsilon*-Pareto Optimal
- Within epsilon of the Pareto Optimal points
- It does *not* necessarily select the Pareto Optimal points

epsilon-Lexicase Selection



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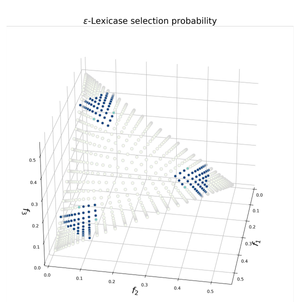
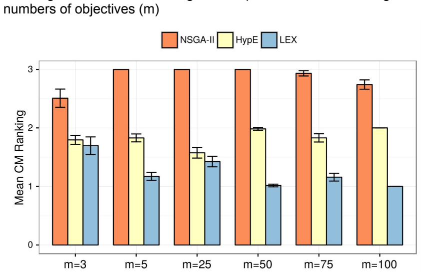
Many objective optimization



Convergence Measure Rankings, DTLZ problems, for increasing numbers of objectives (m)



La Cava, W. & Moore, J. H. (2018) An Analysis of ϵ-Lexicase Selection for Large-Scale Many-Objective Optimization. *GECCO*

Why does Lexicase Selection Work?

Specialists vs. Generalists

❖ **Specialists**:
- relatively low errors on a subset of training cases
- relatively high errors on other training cases
- poor total error (aggregate fitness) relative to population

Low (good) errors on green cases

Ex: 8 9 1 8 7 0 0 7 9 = 49 total

High (bad) errors on red cases

❖ **Generalists**:
- similar errors on all training cases
- not particularly low errors on any training cases
- good total error relative to population

Mediocre errors on all cases

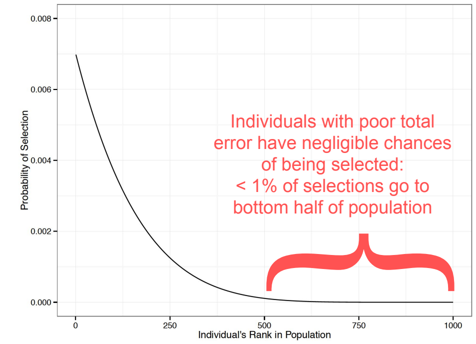
Ex: 3 2 3 3 3 3 4 2 3 = 26 total

Thomas Helmuth et al. (2019) Lexicase selection of specialists. *GECCO*

Specialists vs. Generalists

❖ Which are better to select?
- Aggregating errors emphasizes generalists
- Lexicase selection emphasizes specialists

❖ Empirical answer is specialists in most cases



Ex: Tournament size = 7

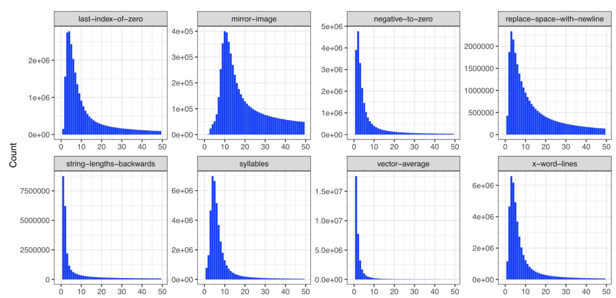
Thomas Helmuth et al. (2019) Lexicase selection of specialists. *GECCO*

Groups of Cases

❖ Most lexicase selection events use small numbers of cases

❖ Cases near the beginning of the shuffled list have largest impact on selection

❖ Cases near end of list have little or no impact!

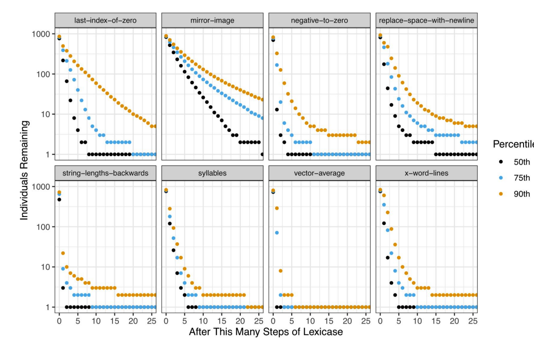


Number of Training Cases used in Selection

Thomas Helmuth, et al. (2020) On the importance of specialists for lexicase selection. *GPEM*

Individuals Remaining after *x* Lexicase steps

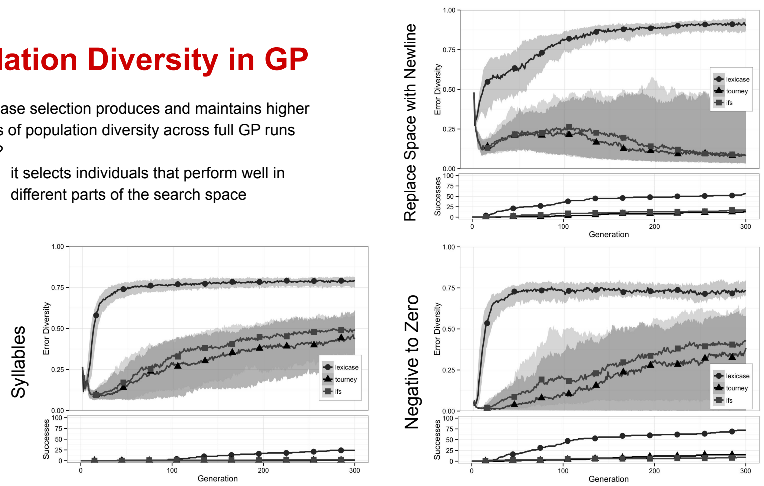
❖ Selection pools often reduce to a small number of individuals within 5-10 cases



Thomas Helmuth, et al. (2020) On the importance of specialists for lexicase selection. *GPEM*

Population Diversity in GP

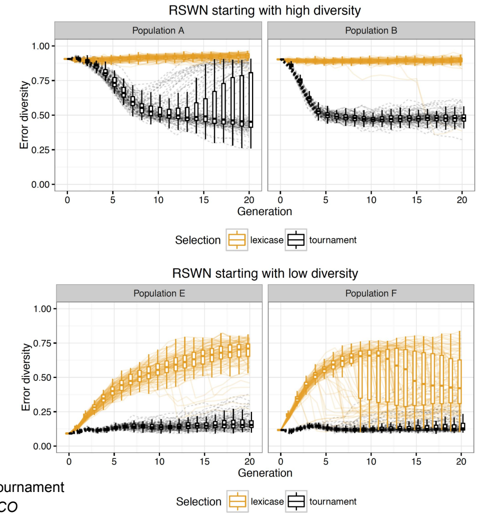
- Lexicase selection produces and maintains higher levels of population diversity across full GP runs
- Why?
  - it selects individuals that perform well in different parts of the search space



Thomas Helmuth et al. (2015) Lexicase selection for program synthesis: A diversity analysis. *GPTP*

Experiment: Diversity in GP

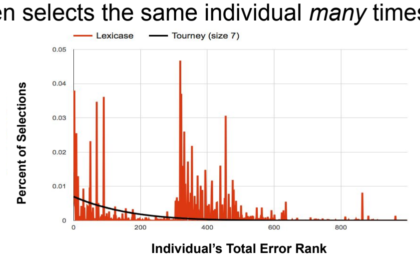
- Start with the same population
  - tested low diversity and high diversity populations
- Measure diversity over next 20 generations
- Tournament selection loses diversity or maintains an initial low diversity
- Lexicase selection maintains an initial high diversity or increases diversity



Thomas Helmuth et al. (2016) Effects of lexicase and tournament selection on diversity recovery and maintenance. *GECCO*

Hyperselection

- Lexicase often selects the same individual *many* times in one generation



- Does this hyperselection help (or hurt) performance?
- Empirical evidence indicates it neither helps nor hurts!

Thomas Helmuth et al. (2016) The impact of hyperselection on lexicase selection. *GECCO*

Running Time

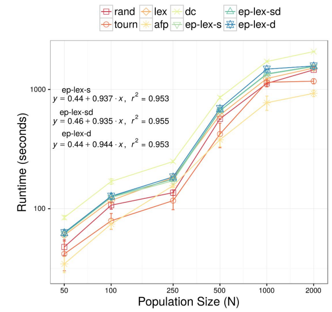
Worst case running time

- Population of N individuals, T training cases
- The worst case running time for a single selection event is $O(NT)$
- For a generation, lexicase selection has worst-case complexity $O(N^2T)$

- Occurs when all individuals are identical
  - In other words, doesn't occur with pre-selection filtering
- Rarely observed

Experimental Running Time

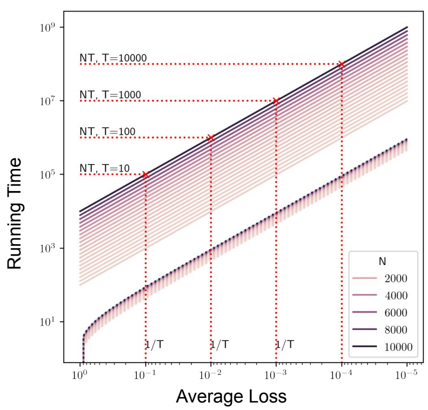
- Observed running time is much better than the worst-case
- Closer to linear in population size



La Cava, W. et al. (2019). A Probabilistic and Multi-Objective Analysis of Lexicase Selection and Epsilon-Lexicase Selection. *Evolutionary Computation*

What about expected running time?

Under some assumptions, we can show that the expected running time of lexicase selection grows *linearly* with the population's average loss, approaching the worst case as the population converges.



Helmuth, T. & La Cava, W. (2021) Expected Running Time of Lexicase Selection. *Under Review*

Extensions

Extensions

❖ Alternate definitions of epsilon
  - User-defined thresholds
    - Moore & McKinley (2016) A Comparison of Multiobjective Algorithms in Evolving Quadrupedal Gaits. *SAB*
    - La Cava et al (2016) Epsilon lexicase selection for regression. *GECCO*
  - MADCAP epsilon lexicase
    - Spector, L. et al. (2018) Relaxations of Lexicase Parent Selection. *GPTP XV*

❖ epsilon-lexicase survival
  - La Cava, W.; Moore, J. (2017) A General Feature Engineering Wrapper for Machine Learning Using epsilon-Lexicase Survival. *EuroGP*

❖ Combinations with other methods
  - Novelty search: Knobelty and novelty-lexicase
  - DOCLEX
    - Liskowski, P.; Krawiec, K. (2017) Discovery of Search Objectives in Continuous Domains. *GECCO*

❖ Using smaller pools / islands
  - Works when less selection pressure is desirable

Discovery of Objectives + Lexicase Selection

- Apply clustering to population semantics to identify sub-tasks
- Feed these into lexicase selection

	Individual Errors				
Case	A	B	C	D	E
1	10	8	73	15	15
2	5	7	60	12	12
3	5	8	0	14	0
4	15	8	0	15	106
5	10	7	1	1	1

→

	Clustered Errors				
Cluster	A	B	C	D	E
1	-	-	-	-	-
2	-	-	-	-	-
3	-	-	-	-	-

→ Lexicase selection

Liskowski, P.; Krawiec, K. (2017) Discovery of Search Objectives in Continuous Domains. *GECCO 17*

Down-sampled Lexicase Selection

❖ Each generation, use a subsample of the training cases to evaluate individuals
  - Similar to mini-batches used in gradient descent

❖ Fewer program evaluations → longer evolution for the same computational cost

❖ Works very well, even using small portions (5-10%) of the training set

- Hernandez, J. G. et al. (2019). Random subsampling improves performance in lexicase selection. *GECCO.*
- Ferguson, A. J. et al. (2019). Characterizing the Effects of Random Subsampling on Lexicase Selection. *GPTP.*
- Thomas Helmuth and Lee Spector. (2020) Explaining and exploiting the advantages of down-sampled lexicase selection. *ALife.*

Weighted Case Shuffling

❖ Natural question: is there a better way to shuffle cases than uniformly random?

❖ Tested:
  - 3 different weighted shuffle algorithms
  - 9 different bias metrics for weighting cases

❖ None of these outperform uniform shuffle!

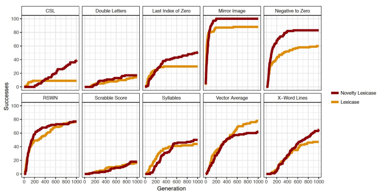
❖ Why? Hypotheses:
  - Lower diversity because of less even emphasis on the search space
  - Fewer selections of specialists that perform well on cases that receive less emphasis

Sarah Anne Troise, Thomas Helmuth. (2017) Lexicase selection with weighted shuffle. *GPTP.*

Combining Lexicase and Novelty Search

Novelty Lexicase Selection

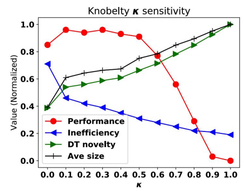
- Combines novelty scores on each case and errors into one set of cases
- Produces more diversity and higher successes in long GP runs



Lia Jundt, Thomas Helmuth. (2019). Comparing and combining lexicase selection and novelty search. *GECCO.*

Knobelty

- Uses novelty search selection K proportion of the time and lexicase selection (1 - K) proportion of the time



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Live Demo

Acknowledgments

- Lee Spector, Nic McPhee, Bill Tozier, Jason Moore
- Epistasis Lab at UPenn
- Grants
  - La Cava: NIH K99-LM012926

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